

Vertical operators on the Bergman space over the upper half-plane: Integral representation

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Let $\mathcal{A}^2(\Pi)$ be the Bergman space of all holomorphic functions on the upper half-plane Π which are square integrable with respect to the Lebesgue measure. This space is a reproducing kernel Hilbert space and every bounded operator T on $\mathcal{A}^2(\Pi)$ can be uniquely written as an integral operator of the form

$$(Tf)(z) = \int_{\Pi} f(w)A_T(z, \bar{w})dw, \quad z \in \Pi, \quad (1)$$

where $A_T(z, \bar{w}) := \overline{(T^*K_{\Pi,z})(w)} = \overline{\langle T^*K_{\Pi,z}, K_{\Pi,w} \rangle_{\mathcal{A}^2}} = \overline{\langle K_{\Pi,z}, TK_{\Pi,w} \rangle_{\mathcal{A}^2}} =: \overline{A_{T^*}(w, \bar{z})}$.

In [2], vertical operators are introduced. In this talk, we discuss integral representation of the form (1) for vertical operators. As a consequence, we study various operator theoretic properties of these operators and we also see the integral representation of the form (1) for the operators in the C^* -algebra generated by Toeplitz operators $T_{\mathbf{a}}$ with vertical defining symbols ($\mathbf{a} \in L^\infty(\Pi)$), which are considered in [2].

This talk is based on the joint work with D. Venku Naidu and P Mohan.

[1] Shubham R. Bais, D. Venku Naidu and P. Mohan, Integral representation of vertical operators on the Bergman space over the upper half-plane, C. R. Math. Acad. Sci. Paris, Accepted for publication (2023)

[2] C. Herrera Yañez, E. A. Maximenko and N. Vasilevski, Vertical Toeplitz operators on the upper half-plane and very slowly oscillating functions, Integral Equations Operator Theory **77** (2013), no. 2, 149–166.