TOEPLITZ OPERATORS ASSOCIATED TO DILATIONS ON THE BERGMAN SPACE OF THE SIEGEL DOMAIN

ARMANDO SÁNCHEZ NUNGARAY, THIS IS A JOINT WORK WITH RAÚL QUIROGA BARRANCO

In this talk, we introduce a coordinate system for each Maximal abelian subgroup (MASG) in the D (unit ball or Siegel Domain), these system are given by $(h,g) \in H(D) \times G$ which depend of the action of the MASG G and h the Moment map associated to the MASG. Using the systems of coordinates associated to Quasi-Elliptic and Quasi-Hiperbollic MASG, we introduce an isometric isomorphism

$$R : L_2(\mathbb{R}) \otimes L_2(\mathbb{B}^{n-1}, d\mu_\lambda) \to L_2(D_n, d\mu_\lambda).$$

such that

$$R(L_2(\mathbb{R}) \otimes \mathcal{A}^2_\lambda(\mathbb{B}^{n-1})) = \mathcal{A}^2_\lambda(D_n),$$

where $\mathcal{A}^2_\lambda(\mathbb{B}^{n-1})$ and $\mathcal{A}^2_\lambda(D_n)$ denote the Bergman space on the ball and Siegel domain.

Furthermore

$$\begin{aligned} R\left(I \otimes B_{\mathbb{B}^{n-1},\lambda}\right) R^* = B_{D_n,\lambda} &: \quad L_2(D_n, d\mu_\lambda) \longrightarrow \mathcal{A}^2_\lambda(D_n), \\ R^* B_{D_n,\lambda} R = I \otimes B_{\mathbb{B}^{n-1},\lambda} &: \quad L_2(\mathbb{R}) \otimes L_2(\mathbb{B}^{n-1}, d\mu_\lambda) \longrightarrow L_2(\mathbb{R}) \otimes \mathcal{A}^2_\lambda(\mathbb{B}^{n-1}), \end{aligned}$$

where $B_{\mathbb{B}^{n-1},\lambda}$ is the Bergman projection from $L_2(\mathbb{B}^{n-1}, d\mu_\lambda)$ onto $\mathcal{A}^2_\lambda(\mathbb{B}^{n-1})$ and $B_{D_n,\lambda}$ is the Bergman projection from $L_2(D_n, d\mu_\lambda)$ onto $\mathcal{A}^2_\lambda(D_n)$.

Here we consider four types of symbols invariant under dilations ($r_0 \cdot z = (r_0^{\frac{1}{2}} z', r_0 z_n)$. where $r_0 \in \mathbb{R}_+$.)

$$a = a(h_n) \in L_\infty(\mathbb{R}) \tag{1}$$

$$a = a\left(\frac{2h_n}{1 + \|h'\|}\right) \in L_\infty(\mathbb{R}) \tag{2}$$

$$b = b(t', h') \in L_\infty(\mathbb{T}^{n-1} \times \mathbb{R}^{n-1}_+) \bigcap L_\infty(D_n)^{\mathbb{T}}. \tag{3}$$

$$c = c(t', h) \in L_\infty(\mathbb{T}^{n-1} \times \mathbb{R}^n_+ \times \mathbb{R}) \bigcap L_\infty(D_n)^{\mathbb{T}}. \tag{4}$$

where $h = (h', h_n) \in \mathbb{R}^{n-1}_+ \times \mathbb{R}$ and $\bigcap L_\infty(D_n)^{\mathbb{T}}$ are function invariant under the action $t_0 \cdot (z', z_n) = (t_0 z', z_n)$.

We obtain the the Toeplitz operators for each of the above symbols are unitary equivalent to the following

$$R^* T_a R = \bigoplus_{k\in\mathbb{Z}_+} \int_{\mathbb{R}}^{\oplus} T_{\hat{a}[k,\xi]} \mid_{\mathcal{H}_k} d\xi \tag{5}$$

$$R^* T_a R = \bigoplus_{k\in\mathbb{Z}_+} \gamma_a(k,\xi) I \tag{6}$$

$$R^* T_c R = \bigoplus_{k\in\mathbb{Z}_+} \int_{\mathbb{R}}^{\oplus} T_{\hat{c}[k,\xi]} \mid_{\mathcal{H}_k} d\xi \tag{7}$$

$$R^* T_b R = \bigoplus_{k\in\mathbb{Z}_+} \int_{\mathbb{R}}^{\oplus} T_b \mid_{\mathcal{H}_k} d\xi = \int_{\mathbb{R}}^{\oplus} T_b d\xi, \tag{8}$$

acting on

$$L_2(\mathbb{R}) \otimes \mathcal{A}^2_\lambda(\mathbb{B}^{n-1}) = \bigoplus_{k\in\mathbb{Z}_+} (L_2(\mathbb{R}) \otimes \mathcal{H}_k) = \bigoplus_{k\in\mathbb{Z}_+} \left(\int_{\mathbb{R}}^{\oplus} \mathcal{H}_k d\xi \right).$$

Finally, we obtain some commutative properties between the above operators, for example: Consider the symbols a, and b and c of the form (**??**), (**??**) and (**??**) respectively. Then $T_aT_c = T_cT_a$. and $T_{ab} = T_aT_b = T_bT_a$.