Compact Toeplitz and Hankel operators on true polyanalytic Fock spaces

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Consider the Gaussian measure μ given by $\mathrm{d}\mu = \frac{1}{\pi}e^{-|z|^2}\,\mathrm{d}z$. Then the standard (analytic) Fock space $\mathcal{F}^2$ consists of functions $f \in L^2(\mathbb{C},\mu)$ that satisfy $\frac{\partial f}{\partial \bar{z}} = 0$. If we replace the $\bar{\partial}$-operator by $\bar{\partial}^n$, we obtain the polyanalytic Fock space $\mathcal{F}_n^2$, that is,

$$\mathcal{F}_n^2 := \{f \in L^2(\mathbb{C},\mu) : \tfrac{\partial^n f}{(\partial \bar{z})^n} = 0\}.$$

The so-called true polyanalytic Fock spaces are then constructed as follows:

$$\mathcal{F}_{(1)}^2 := \mathcal{F}^2, \quad \mathcal{F}_{(n)}^2 := \mathcal{F}_n^2 \ominus \mathcal{F}_{n-1}^2, \quad n \geq 2.$$

In this talk I will explain how limit operator methods [2] can be used to characterize the set of compact operators on $\mathcal{F}_{(n)}^2$. In fact, with the help of the decomposition

$$L^2(\mathbb{C},\mu) = \bigoplus_{n=1}^{\infty} \mathcal{F}_{(n)},$$

we will obtain a characterization of compact operators on $L^2(\mathbb{C},\mu)$. A straightforward argument then shows that a Hankel operator with bounded symbol f,

$$H_{f,(n)} : \mathcal{F}_{(n)}^2 \to L^2(\mathbb{C},\mu),$$

is compact if and only if f has vanishing mean oscillation. In particular, this means that the compactness of $H_{f,(n)}$ is independent of n. Interestingly, the same cannot be said about Toeplitz operators

$$T_{f,(n)} : \mathcal{F}_{(n)}^2 \to \mathcal{F}_{(n)}^2.$$

This talk is an extended version of my presentation at IWOTA 2022 in Krakow, and based on the paper [1].

[1] R. Hagger: *Toeplitz and related operators on polyanalytic Fock spaces*, preprint on arXiv: 2201.10230.

[2] R. Hagger, C. Seifert: *Limit operators techniques on general metric measure spaces of bounded geometry*, J. Math. Anal. Appl. 489 (2020), no. 2, 124180, 36 pp.